

FEB 17 1947

ARR May 1942

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

# WARTIME REPORT

ORIGINALLY ISSUED

May 1942 as  
Advance Restricted Report

A FLIGHT INVESTIGATION OF SHORT-PERIOD

LONGITUDINAL OSCILLATIONS OF AN

AIRPLANE WITH FREE ELEVATOR

By William H. Phillips

Langley Memorial Aeronautical Laboratory  
Langley Field, Va.

FOR REFERENCE

NOT TO BE TAKEN FROM THIS ROOM

# NACA

WASHINGTON

NACA LIBRARY  
LANGLEY MEMORIAL AERONAUTICAL  
LABORATORY  
Langley Field, Va.

NACA WARTIME REPORTS are reprints of papers originally issued to provide rapid distribution of advance research results to an authorized group requiring them for the war effort. They were previously held under a security status but are now unclassified. Some of these reports were not technically edited. All have been reproduced without change in order to expedite general distribution.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

ADVANCE RESTRICTED REPORT

A FLIGHT INVESTIGATION OF SHORT-PERIOD  
LONGITUDINAL OSCILLATIONS OF AN  
AIRPLANE WITH FREE ELEVATOR

By William H. Phillips

SUMMARY

A flight investigation has been made to check the results of a theoretical analysis of longitudinal stability of an airplane with free controls. Tests were made of a Fairchild XR2K-1 airplane on which the weight moment and the aerodynamic balance of the elevator were varied to bring it into a condition where unstable short-period oscillations were encountered. The amounts of aerodynamic balance and weight moment required for instability were found to be greater than the amounts predicted by the theory. Time histories of the oscillations are included to show the nature of the instability.

INTRODUCTION

During tests made to determine the flying qualities of airplanes of different types, several airplanes exhibited a form of instability in which oscillations of the airplane were coupled with oscillations of the control surfaces through the mass unbalance of moving parts in the control system. This condition had a very undesirable effect on the handling characteristics of the airplanes in rough air and was dangerous in the more violent cases because large accelerations could be built up very rapidly. In order to find the design features of the airplane and of the control system responsible for these unstable oscillations, a theoretical study of the stability of an airplane with controls free was made. (See reference 1.) In this investigation the conditions for longitudinal stability with elevator free and for lateral stability with rudder or ailerons free were determined.

An experimental investigation to check the results of the mathematical work was then undertaken. Tests were made

L-444

of a Fairchild XR2K-1 airplane on which the aerodynamic balance and the weight moment of the elevator were varied to bring the airplane into an unstable condition. This report includes the results of these tests and a comparison of these results with the theoretical predictions.

# SYMBOLS

$V_o$	study-flight speed
$\alpha$	angle of attack
$W$	airplane weight
$l$	tail length of airplane
$\theta$	angle of pitch
$M$	pitching moment
$C_m$	pitching-moment coefficient $\left(\frac{M}{q c S}\right)$
$Z$	force along normal axis
$X$	force along longitudinal axis
$H$	elevator hinge moment
$\delta$	elevator deflection
$C_h$	hinge-moment coefficient $\left(\frac{H}{S_e \frac{\rho}{2} V_o^2 c_e}\right)$
$S$	wing area
$c$	wing chord
$S_e$	elevator area
$c_e$	elevator chord
$\rho$	air density
$q$	dynamic pressure

$x_{a.c.}$	projection on X axis of distance between center of gravity of airplane and its aerodynamic center.
$\bar{x}_e$	moment arm of center of gravity of elevator about hinge axis, positive when center of gravity is behind hinge.
$k_e$	radius of gyration of elevator mechanism about elevator hinge axis
$k_Y$	radius of gyration about Y axis
$\mu$	airplane density ratio $\left( \frac{m}{S \frac{\rho}{2} c} \right)$
$m$	mass of airplane
$\mu_e$	elevator density ratio $\left( \frac{m_e}{S_e \frac{\rho}{2} c_e} \right)$
$m_e$	mass of elevator
$a_n$	normal acceleration $(\dot{w} - V_o \dot{\theta})$
$w$	normal component of velocity
$\dot{w}$	$= \frac{dw}{dt}$
$\dot{\theta}$	$= \frac{d\theta}{dt}$
$\dot{\delta}$	$= \frac{d\delta}{dt}$
$\ddot{\theta}, \ddot{\delta}, \text{etc.}$	$= \frac{d^2\theta}{dt^2}, \frac{d^2\delta}{dt^2}, \text{etc.}$
$I$	moment of inertia
$C_{h\delta}$	aerodynamic hinge-moment parameter $\left( \frac{\partial C_h}{\partial \delta} \right)$
$C_{hD\delta}$	$= \frac{\partial C_h}{\partial D\delta}$

D operator indicating differentiation with respect  
to distance  $\left(\frac{d}{ds}\right)$

s distance along flight path

A diagram showing the convention of axes used is given in figure 1.

### THEORETICAL INVESTIGATION

In the theoretical study of stability (reference 1), the airplane and control-system characteristics are expressed in terms of nondimensional ratios defined previously in the list of symbols. All distances are expressed in nondimensional form in terms of half-chord lengths of the wing. Quantities are differentiated with respect to distance rather than time. Stability derivatives depending on the rate of change of a quantity must therefore be computed with the aid of the formula for the differential operator:

$$D = \frac{d}{ds} = \frac{c/2}{V_0} \frac{d}{dt}$$

For example,

$$C_{hDs} = \frac{dC_h}{d\delta} = \frac{V_0}{c/2} \frac{dC_h}{d\delta}$$

The following parameters are found to be important in determining the stability of the motion.

$\mu_e \bar{x}_e$  elevator mass-unbalance parameter

$\mu_e k_e^2$  elevator moment-of-inertia parameter

The solution of the equations of motion shows the existence of two modes of oscillation. One mode is well-damped and the other, which involves reinforcement of the pitching motion of the airplane by the flapping of the elevator, is likely to be poorly damped or unstable if the elevator has a large weight unbalance, a high degree of aerodynamic balance, or a large moment of inertia. Curves

showing the limiting values of these quantities allowable for stability are given in figure 2, which has been adapted from reference 1. In this figure, any design that falls to the left of the boundary of stability defined by its particular value of  $\mu_{ek}e^2$ , in the area labeled "stable region" will theoretically be stable, while a design that gives a point to the right of this boundary will experience unstable oscillations. The "region of divergence" on the left side of figure 2 shows that designs on which the elevator center of gravity is ahead of the hinge line may experience instability in the form of a rapid divergence. Because this type of instability was not the subject of the present investigation, the reader is referred to reference 1 for a more complete explanation.

Boundaries for stability were obtained in the investigation by assuming typical values of airplane density, moment of inertia, and aerodynamic derivatives. None of these characteristics were found to have any large effect on the stability of the oscillation when they were varied within the range customarily used in airplane design. The only factor, other than those mentioned, that is likely to have an appreciable effect on the oscillation is the damping of the elevator motion  $C_{h\dot{\delta}}$ . This factor is difficult

to evaluate, because it includes friction in the elevator system. Its effect will be discussed more fully later.

### EXPERIMENTAL INVESTIGATION

In order to check the boundaries of stability computed from the theory by actual flight tests, the elevator system of the XR2K-1 airplane was linked to a pivoted rod to which movable lead weights were attached. This rod was placed within reach of the pilot so that the moment of inertia and the mass unbalance of the elevator system could be varied during flight.

A description of the XR2K-1 airplane is given in the appendix. A photograph of the airplane is shown in figure 3. Figure 4 shows the linkage used to attach the lead weights to the elevator-control system.

The program of flight tests included measurements of the elevator force and position at various airspeeds and with three positions of the center of gravity of the airplane. From these measurements the elevator restoring-moment coefficient  $C_{h\delta}$  could be computed. The elevator

mass unbalance and the moment of inertia were then varied into the range shown by the theory to cause instability with the value of  $C_{h\delta}$  which was found to exist.

When the flight tests were made, however, it was found that no unstable oscillations were obtained at the boundaries indicated in figure 2. Vibrations in the elevator motion introduced by abruptly deflecting the control stick and releasing it were damped out almost immediately. In an effort to reach an unstable condition, a balance tab with variable linkage was placed on the elevator. By the use of this tab to reduce the elevator restoring-moment coefficient  $C_{h\delta}$  to a very small value, unstable oscillations could be obtained in the higher part of the speed range. The stable region thus defined, however, was much broader than the region indicated by figure 2.

Two factors might have caused the unexpected stability of the elevator motion: The location of the weights used to unbalance the elevator near the cockpit instead of on the elevator itself, and the existence of a large amount of friction in the elevator-control system.

Further tests were made with the weighted rod removed from the cockpit and with a lead weight attached to the elevator behind the hinge line. Theoretical considerations indicated and the flight tests verified that instability occurred in the same region as before.

The friction force in the elevator system, however, was known to be larger than the value assumed in the theory. Figure 2 was plotted with an assumed value of 1.0 for  $C_{h\delta}$ , which is the theoretical value for the aerodynamic damping force on a 50-percent-chord flap. In addition, in the actual control system, a fairly large amount of frictional damping was present. This frictional force was probably partly of the viscous type, which increases in proportion to the angular velocity of the elevator, and partly solid friction, which gives a constant force opposing the motion of the elevator. The solid-friction force is difficult to account for in the theory. All the inertia forces, restoring forces, and viscous damping forces increase directly with the amplitude of the oscillation and are, therefore, small for small vibrations. Thus, a solid-friction force, however small, will prevent motion of the elevator until a certain amplitude is exceeded and will assure the stability of oscillations of this amplitude. When a large solid-friction force exists, as in the case of the Fairchild elevator system, it may be necessary to start the oscillation at a

large amplitude in order to obtain instability. The limit of linearity in the other coefficients may then be exceeded and, as a result, the theory will not correctly explain the observed results.

1-444

In order to obtain a more accurate picture of the nature of forces acting on the control system during an oscillation, records were taken of normal acceleration, control position, pitching velocity, and airspeed. The required forces can be computed from these records by using the following method. The equation of motion for the elevator when it is subjected to the forces resulting from an oscillation of the airplane is:

$$I (\ddot{\delta} + \ddot{\theta}) + m_e \bar{x}_e (a_n + i\ddot{\theta}) = \text{external hinge moment}$$

where the external hinge moment consists of all frictional and aerodynamic forces acting on the elevator. All terms on the left-hand side of the equation may be found from the flight records. The normal accelerations may be obtained directly, the angular velocity of the elevator by differentiating twice the record of the elevator angle, and the pitching acceleration by differentiating the record of pitching velocity. The variation of external hinge moment during an oscillation may then be shown by plotting the values found from the equation.

This procedure has been followed for two runs made with unbalancing weights attached directly to the elevator. An illustration of the graphical work necessary for one of these cases is given in figures 5, 6, and 7. Although graphical differentiation of instrument records is not an exactly accurate process, the results are sufficiently consistent to allow certain conclusions to be drawn. In the first place, because the oscillation of external hinge moment is not in phase with the variation of elevator angle, as shown by a comparison of figures 5(a) and 7, there cannot be a large restoring moment caused by elevator deflection. This conclusion agrees with the pilot's observation that, for the tab-linkage ratio used on this run, the elevator forces were very light. The only other sources of hinge moment are frictional and aerodynamic damping forces. The variation of external hinge moment, therefore, should be in phase with the angular velocity of the elevator. A comparison of figures 5(b) and 7 shows that this condition exists in this case. In fact, the hinge moment as may be seen from figure 8 varies consistently with the angular velocity. If only viscous damping existed, this curve would



be a straight line through the origin. On the other hand, if only solid friction were present, a constant force opposing the motion, independent of angular velocity, would be plotted. The curve actually obtained is approximately a combination of these two types of damping. For the amplitude of the oscillation that existed in this test, an equivalent viscous damping may be obtained by approximating the curve of figure 8 with a straight line. The slope of this line shows a value of  $C_{hD\delta}$  of about -10, or ten times the value assumed in computing the boundaries for stability. Solving the stability equation with this large value for damping shows that the boundary between stable and unstable motion occurs in the same region in which it occurred in the flight tests.

An investigation was made to determine whether the aerodynamic damping force on the elevator of the Fairchild airplane was the same as that predicted by the theory of oscillating airfoils (reference 2). Because the graphical representation of the flight oscillations was not considered sufficiently accurate for this purpose, advantage was taken of an opportunity to measure the elevator damping while the airplane was mounted for other tests in the NACA full-scale tunnel. In these tests the airplane was attached rigidly to the balances in the tunnel. Records were taken of the motion of the elevator when it was fully deflected and then suddenly released. From the rate of decay of the oscillation the total damping in the system was found to give a value of 1.70 for  $C_{hD\delta}$ . Records were then taken of oscillations of the elevator when it was restrained by springs. From these tests, the frictional damping alone could be obtained. The total damping subtracted from the frictional damping gave a value of aerodynamic damping which was in good agreement with the theoretical value of 1.0 for  $C_{hD\delta}$ .

A large discrepancy exists between the value of total damping obtained in the full-scale-tunnel test and the value obtained from the graphical analysis of the flight records. Although, as mentioned previously, the graphical solution may not be very accurate, damping of the oscillations was probably greater in the flight tests than in the full-scale-tunnel tests. The increased damping may possibly be explained by the fact that, in oscillation during flight, a periodic change of downwash from the wing, in phase with the angular velocity of the elevator, opposed the motion.

If the damping of the oscillation in flight was actually lower than the value of -10 for  $C_{hD\delta}$ , indicated by the

graphical analysis, it is necessary to explain why the theory required the use of this large damping moment in order to check the boundaries of stability found in flight. It is possible that the solid friction existing in the control system is much more effective in damping out the motion of the airplane than the equivalent viscous damping determined in a test in which the elevators alone are allowed to move. If this fact is true an equivalent viscous damping cannot be substituted for the effects of solid friction in a motion involving more than one degree of freedom.

## DISCUSSION OF RESULTS

The results of the flight tests of the Fairchild XR2K-1 airplane in longitudinal oscillation may be considered somewhat disappointing, because they do not give a conclusive demonstration that the boundaries for stability of elevator-free motion plotted in figure 2 are the correct ones to use in design.

The curves of figure 2 are plotted for the ideal case in which there is no friction in the elevator system. When friction is present, it will, first of all, always increase the stability of the motion. The boundaries for stability of figure 2 are independent of the speed of the airplane and, as a result, the likelihood of reaching an unstable condition in the ideal case is the same at any flight speed. In an actual control system, however, the frictional damping will be independent of the speed, whereas all the aerodynamic forces will increase as their speed increases. The ratios between frictional force and all the other forces will, therefore, be greater at lower speeds. As a result, instability will first occur at the highest speed of the airplane.

The second effect of friction is to make the stability dependent on the amplitude of the elevator oscillation. When solid friction is present the motion will always be stable below a certain amplitude. It is therefore impossible in this case to plot a boundary for stability that depends only on the control-system characteristics. For motions of large amplitude, however, the frictional forces will be small in comparison with the other forces in the system, so that the ideal case will be more closely approached. It must be remembered that the elevator restoring moment  $C_{h\delta}$  may be much greater for oscillations of large amplitude than

for oscillations of small amplitude, a condition that will tend to increase the stability.

High-speed or large airplanes on which the control forces will ordinarily be excessive are likely to have elevators with sufficient aerodynamic balance to place them in the unstable range. Both of these types are likely to have frictional forces in the control system that are very low in comparison with the aerodynamic forces. The ideal curves will be approached much more closely by the use of airplanes of these designs than by the use of the airplane tested.

Control systems designed to fall within the stable region of figure 2 will always be on the conservative side, because any friction in the system will tend to increase the stability. It is not likely, however, that the design will be too conservative, because both in high-speed flight and with oscillations of large amplitude, the two most dangerous conditions for instability, the effect of friction is minimized in comparison with the aerodynamic forces. Although stability in a control system falling in the unstable region of figure 2 could always be assured by providing enough frictional damping, it is doubtful that this method would ever be practical because of the very undesirable effect that friction has on the "feel" of the controls.

In order to show the severity of the longitudinal oscillations that occur when an unstable condition is reached, time histories of the airspeed, the normal acceleration, the elevator angle, and the pitching velocity during several of the oscillations of the Fairchild airplane are plotted in figures 9 and 10. The oscillations were started by suddenly pulling the stick back and releasing it. The successive records were made as the tab-elevator linkage was progressively increased, thus reducing the restoring moment  $C_{h\delta}$  of the elevator. When the motions became unstable, the normal acceleration oscillated through an amplitude of approximately  $3\frac{1}{2}g$  on the second cycle. When it is realized that these tests were made at the low speeds of 100 and 120 miles per hour, respectively, the danger of an unstable condition existing for an airplane of higher speed is apparent.

Although the longitudinal oscillations treated in this paper are entirely different from flutter, which involves elastic deformation of the structure of the airplane, it may be pointed out that the control-system design charac-

teristics affect the stability of both types of oscillation similarly. If a conventional elevator is mass-balanced to prevent the occurrence of flutter, no trouble should be experienced from control-free oscillations.

L-444

### CONCLUSIONS

1. Violent short-period longitudinal oscillations may occur in an airplane with the elevator control released if the elevator has a high degree of aerodynamic balance and a large mass unbalance.

2. The curves representing the boundaries of elevator-free stability can be used to determine the limits beyond which the mass unbalance, the aerodynamic balance, and the moment of inertia of the elevator control may cause instability. The values determined from these curves are likely to be very conservative for small, low-speed airplanes whose control systems have appreciable friction. The values will probably not be conservative for large or fast airplanes with control systems having a small amount of friction.

Langley Memorial Aeronautical Laboratory,  
National Advisory Committee for Aeronautics,  
Langley Field, Va.

## APPENDIX

## Description of the Fairchild XR2K-1 Airplane

The Fairchild XR2K-1 airplane is a high-wing strut-braced monoplane powered by a 145-horsepower Warner Scarab engine. The principal dimensions of the airplane are as follows:

Span.....	32 ft 10 in.
Over-all length.....	22 ft
Distance from wing aerodynamic center to elevator hinge line.....	15 ft
Wing area.....	171 sq ft
Gross weight.....	1750 lb
Wing loading.....	10.2 lb/sq ft
Stabilizer span.....	9 ft 9 $\frac{1}{2}$ in.
Stabilizer area.....	22.0 sq ft
Elevator area.....	10.4 sq ft

The tail surfaces used in the tests described were not those originally provided with the airplane.

## REFERENCES

1. Jones, Robert T., and Cohen, Doris: An Analysis of the Stability of an Airplane with Free Controls. Rep. No. 709, NACA, 1941.
2. Theodorsen, Theodore: General Theory of Aerodynamic Instability and the Mechanism of Flutter. Rep. No. 496, NACA, 1935.

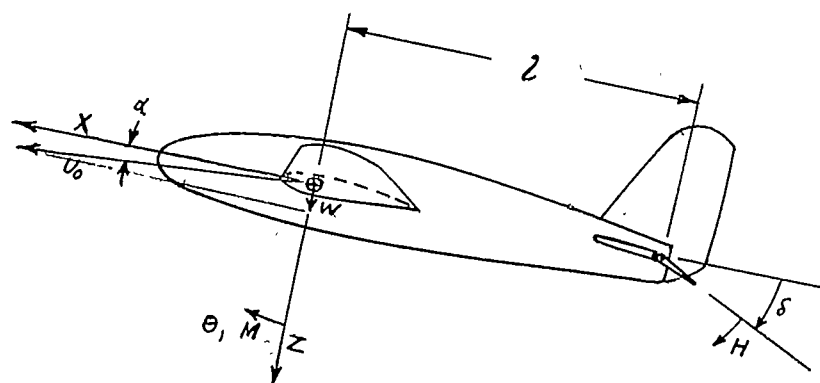


Figure 1.- Notation for longitudinal stability.

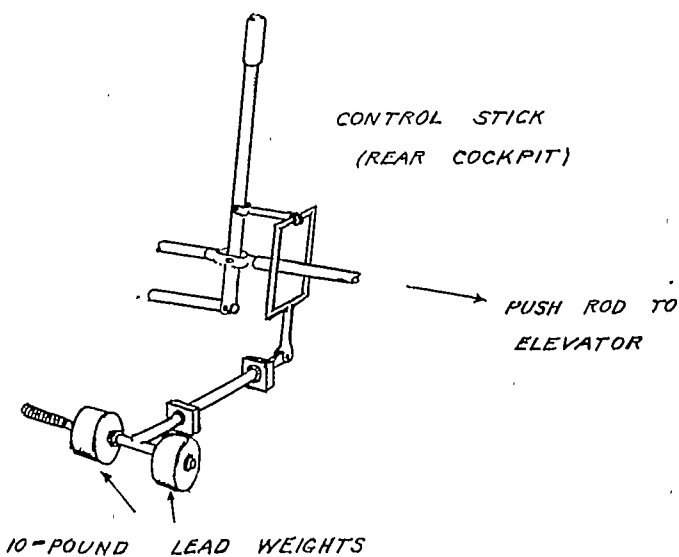


Figure 4.- The method of attaching the lead weights originally used to change the moment of inertia and mass unbalance of the Fairchild XR2K-1 elevator-control system.

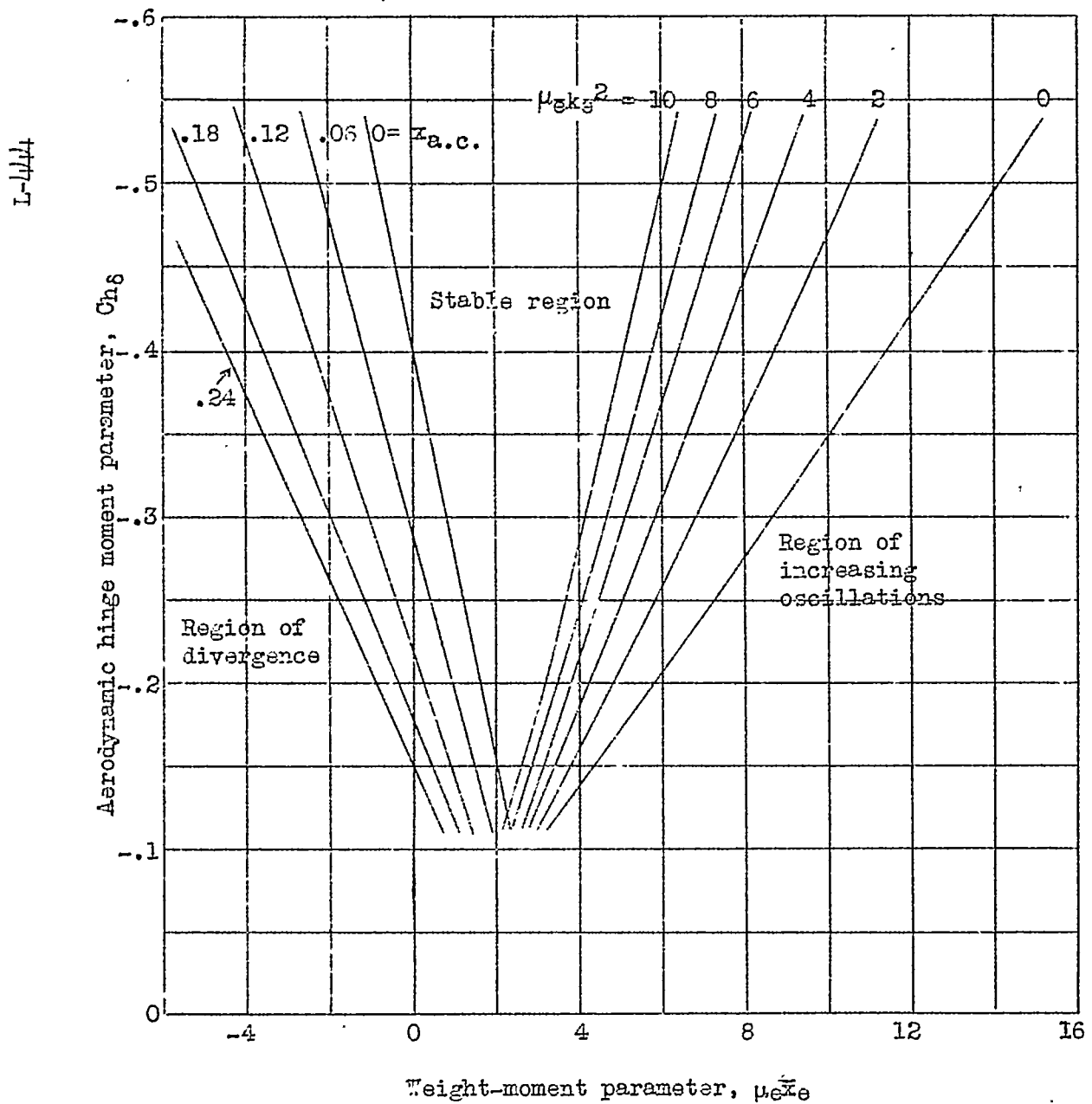


Figure 2.- Boundaries of stability in terms of nondimensional parameters for longitudinal motion with elevator free. 50-percent-chord elevator;  $\mu$ , 45;  $k_y$ , 1.79. (Adaptation of fig. 3 of reference 1.)

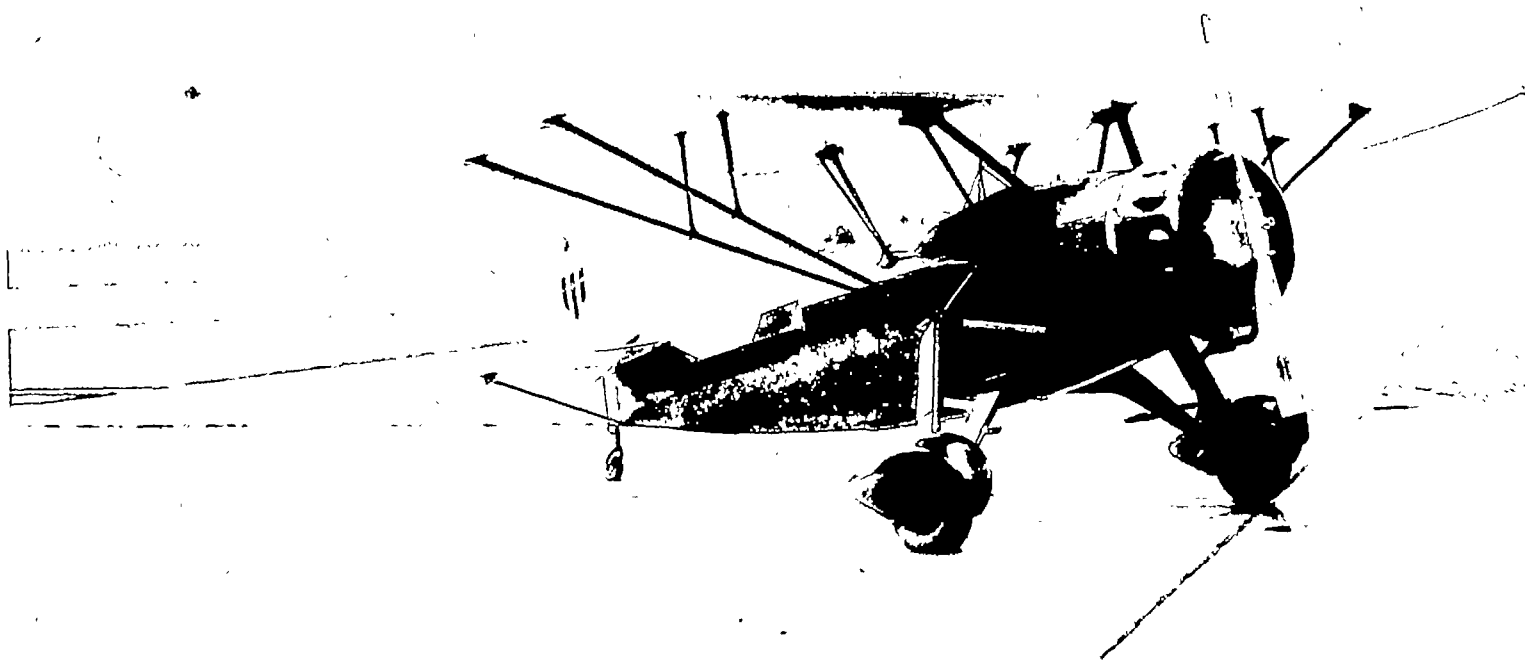


Figure 3.— Fairchild XR2K-1 airplane.



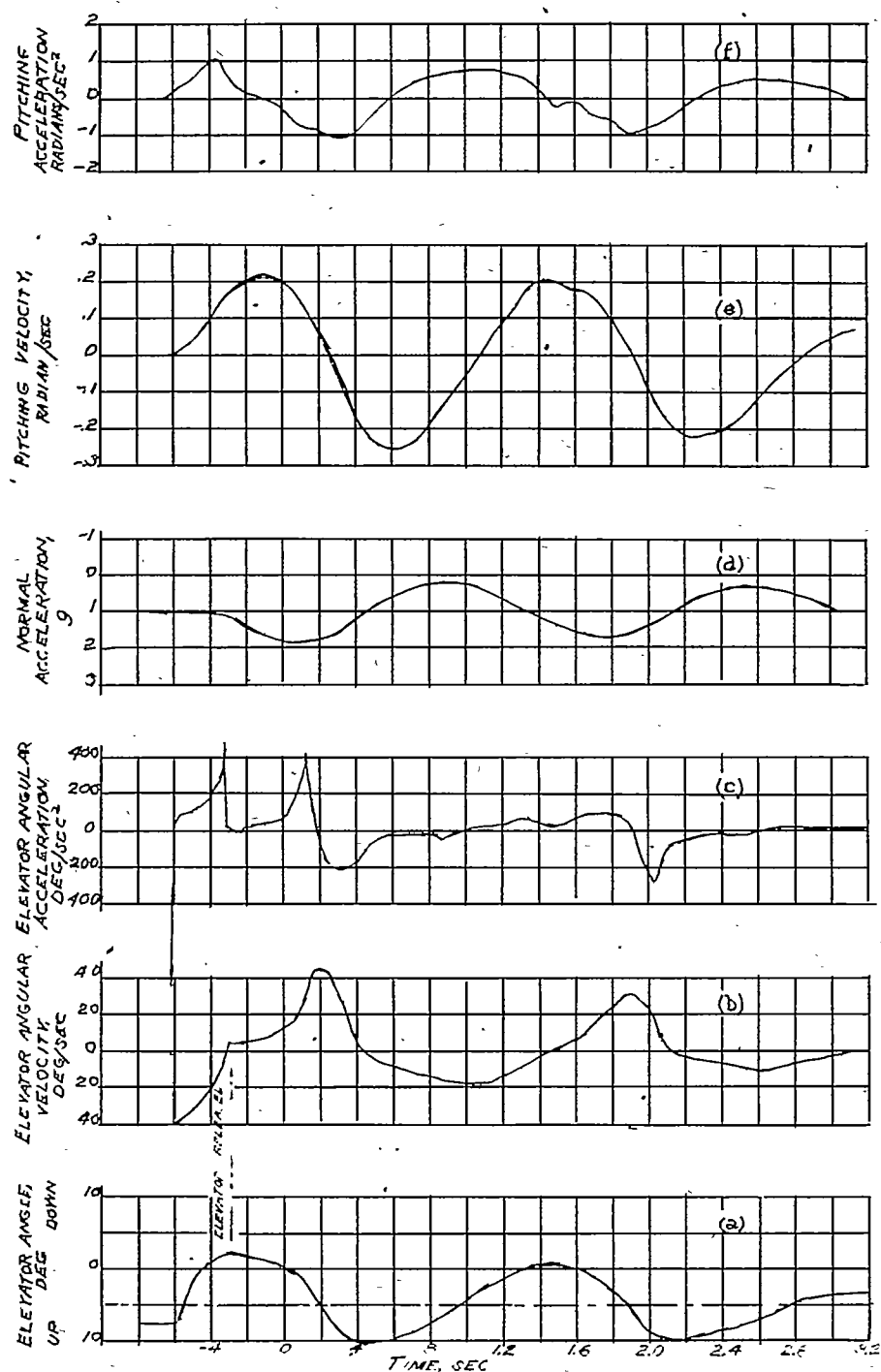


Figure 5.- Variation of normal acceleration, elevator angle, elevator angular velocity, elevator angular acceleration, pitching velocity, and pitching acceleration with time in a control-free longitudinal oscillation. Fairchild XR2K-1 airplane with weighted elevator.

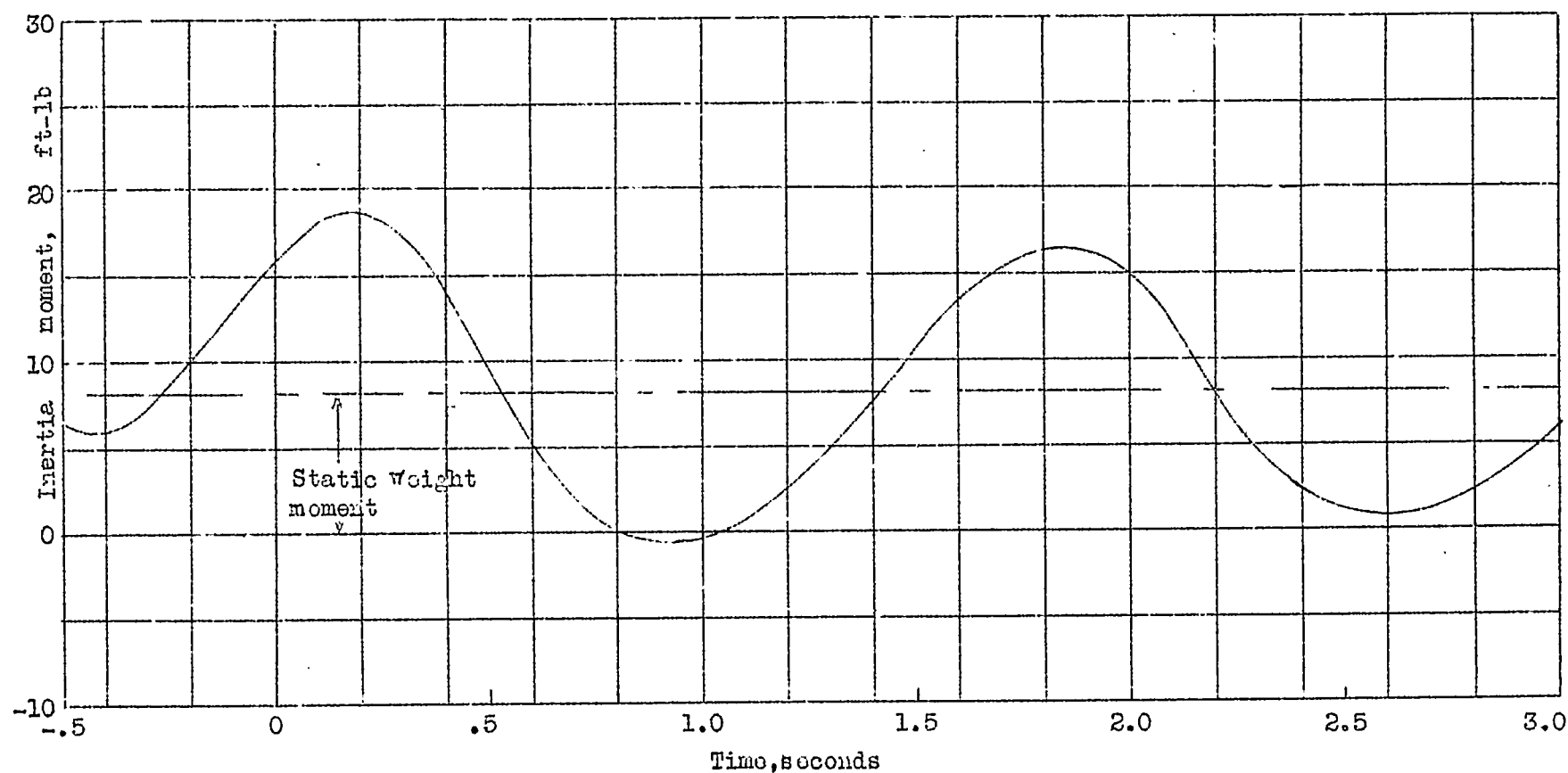


Figure 6.- Variation of inertia moment about elevator hinge caused by unbalance  $m_e \bar{x}_e (\dot{w} - V_0 \dot{\theta} + 2\ddot{\theta})$  with time in control-free longitudinal oscillation. Fairchild XR2K-1 airplane with weighted elevator.

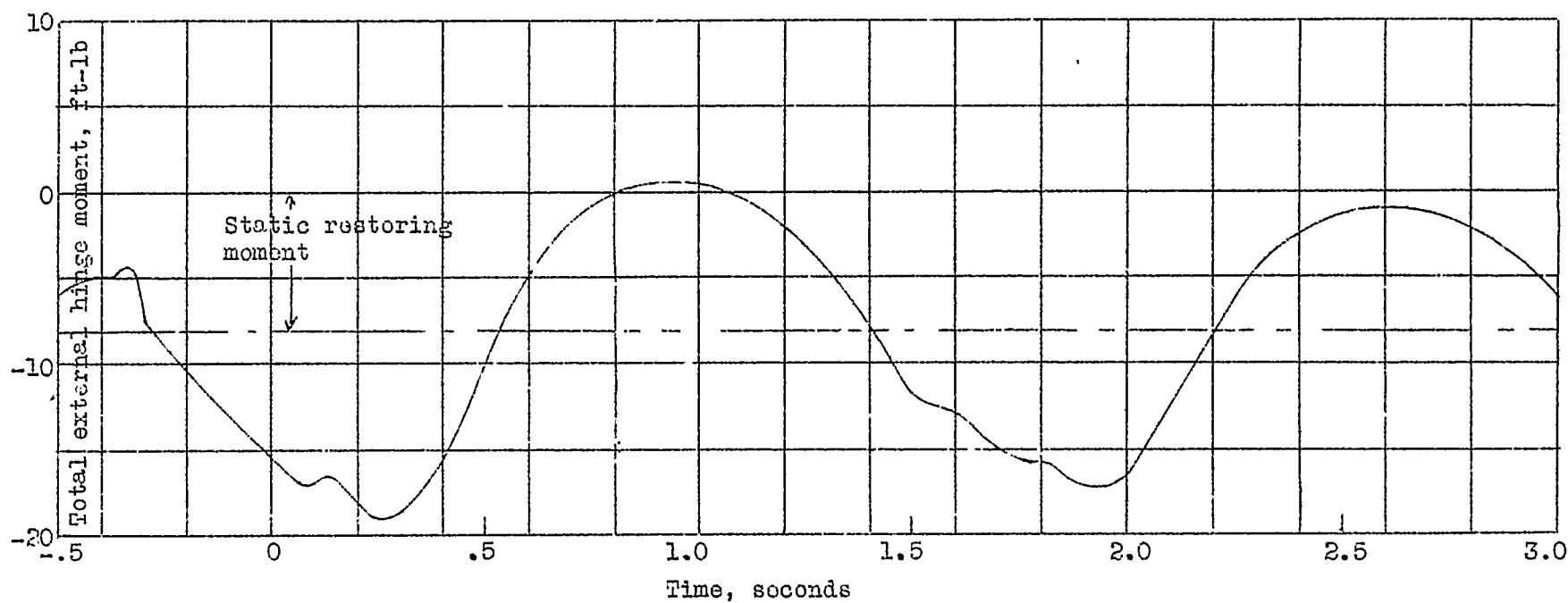


Figure 7.- Variation of total external hinge moment on elevator with time in control-free longitudinal oscillation. Total external hinge moment,  $- [I(\ddot{\delta} + \ddot{\theta}) + m_e \bar{x}_e (\dot{w} - V_e \dot{\theta} + 2\ddot{\theta})]$ . Fairchild XR2X-1 airplane with weighted elevator.

L-444

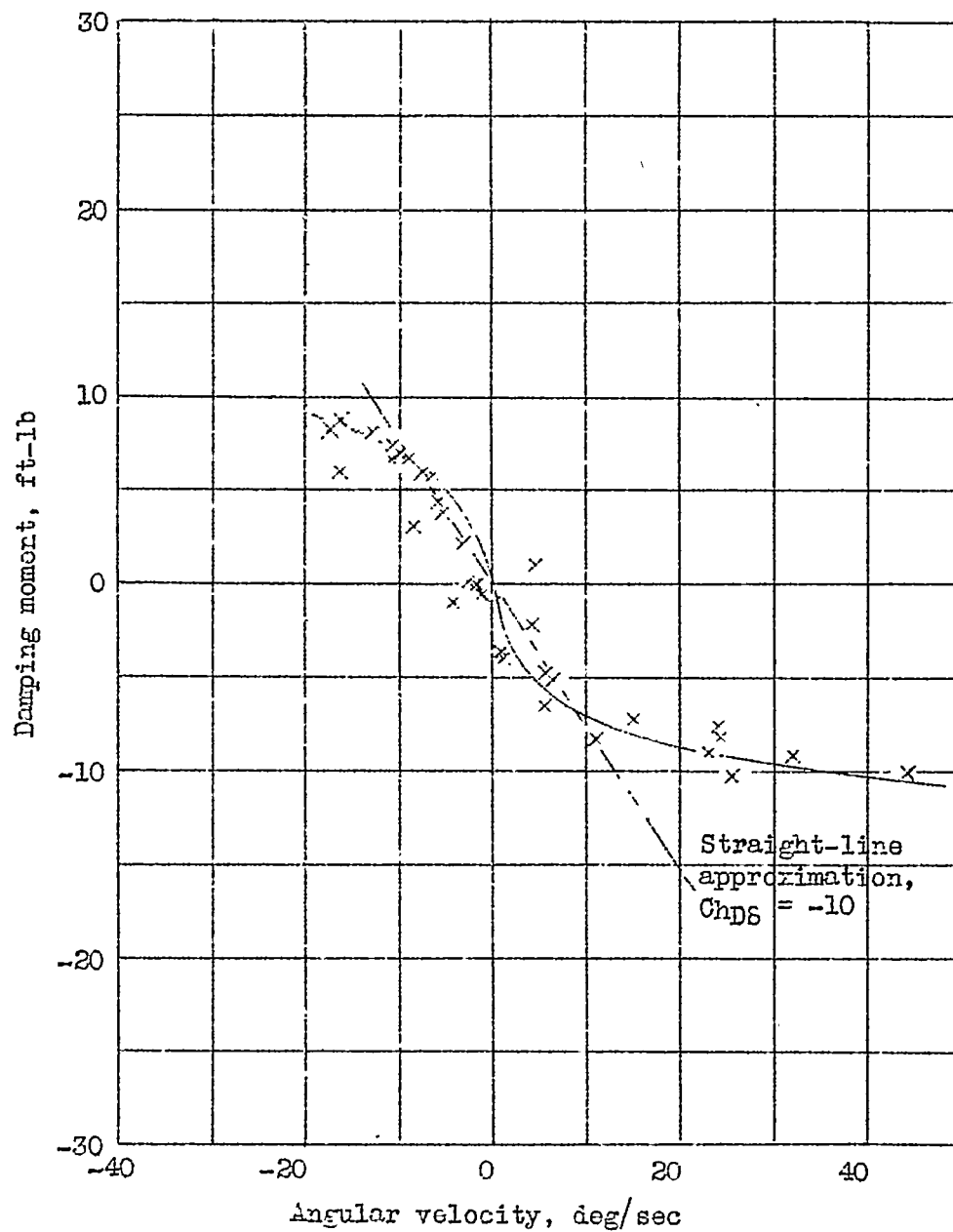


Figure 8.- Variation of damping of elevator motion with angular velocity of elevator during control-free longitudinal oscillation. Fairchild XR2K-1 airplane with weighted elevator.

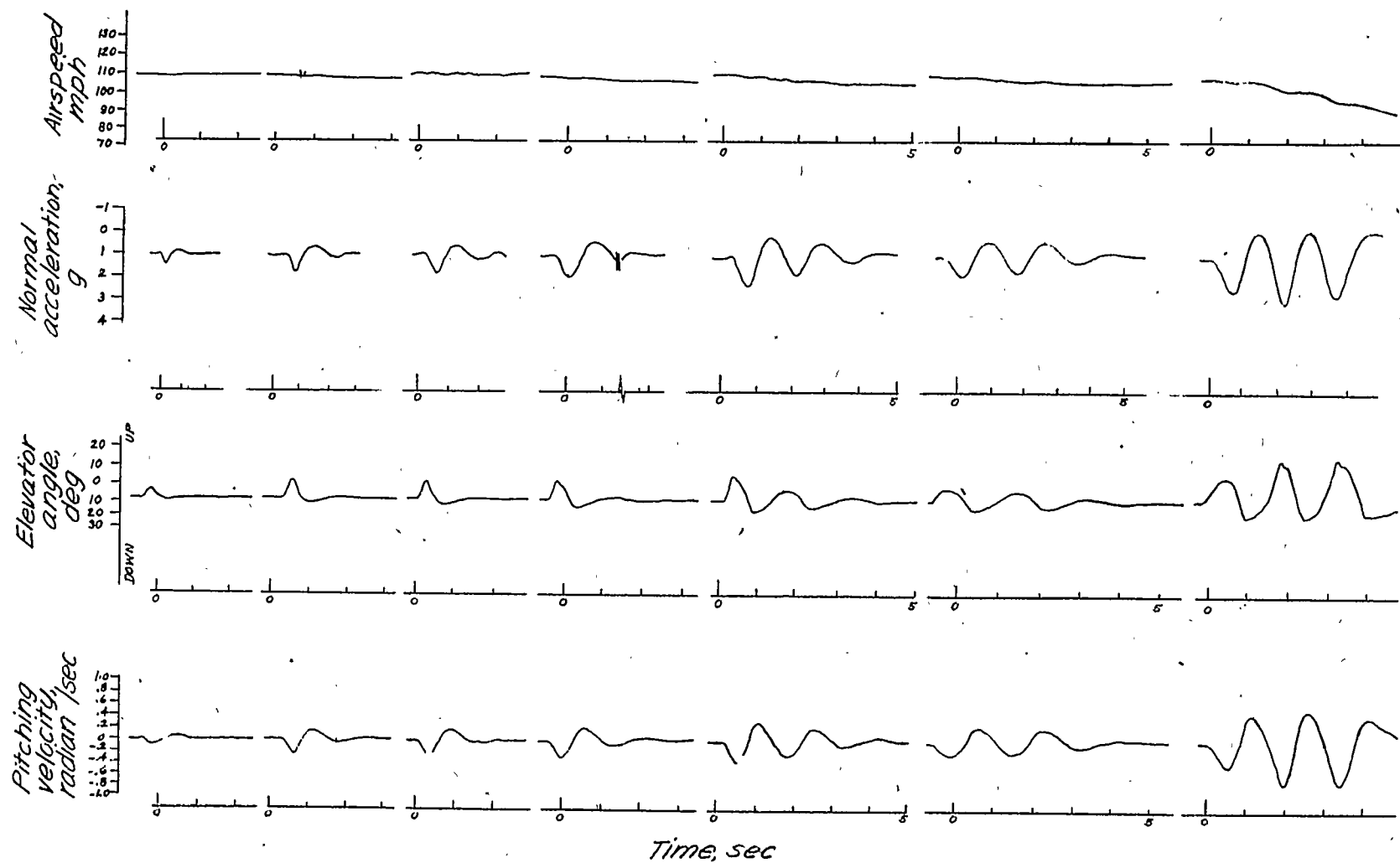


Figure 9.-Time histories of longitudinal oscillations of the Fairchild XR2K-1 airplane with weighted elevator. Aerodynamic balance gradually increased in successive runs. Speed, 100 mph; power on.

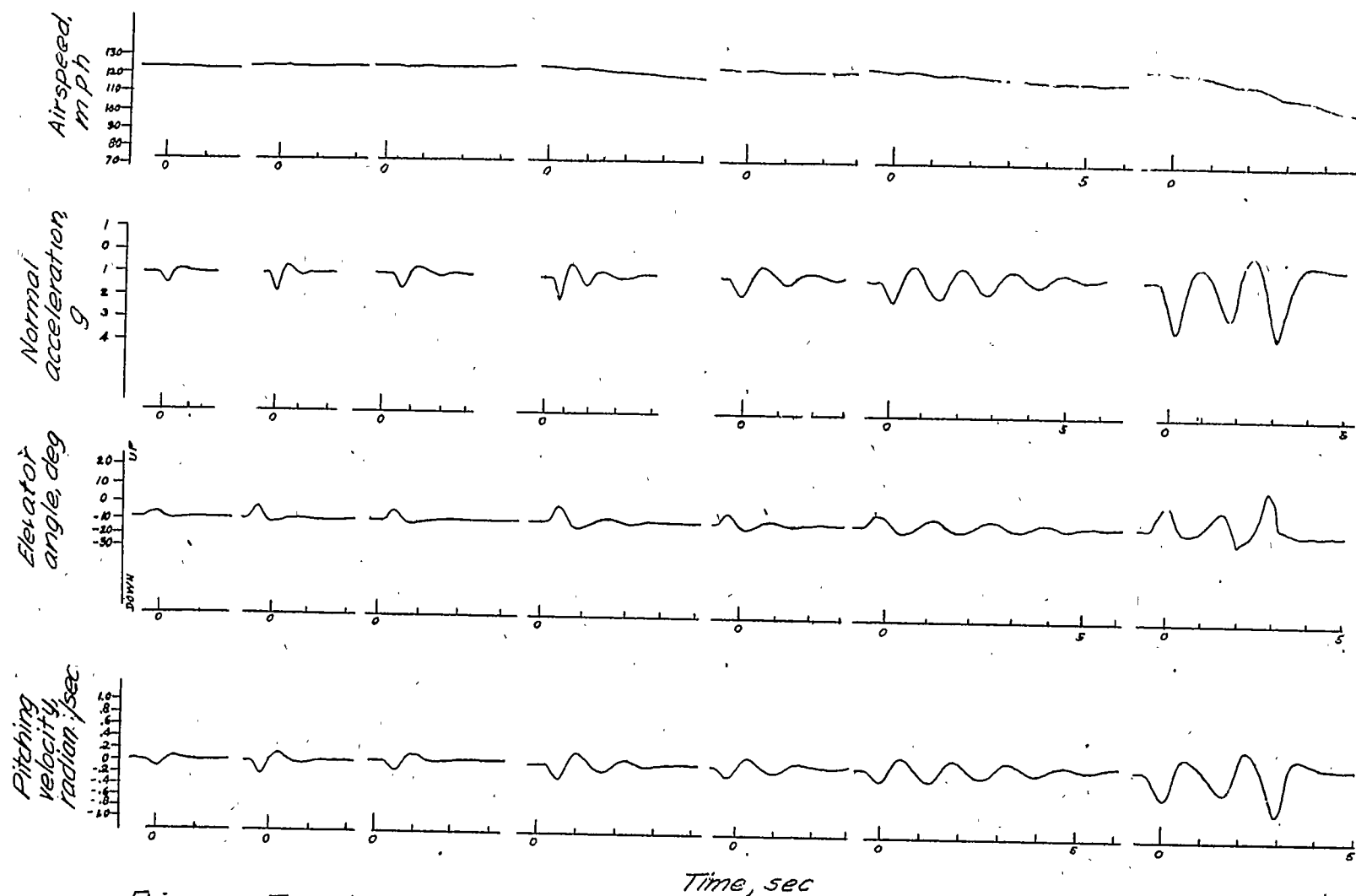


Figure 10.- Time histories of longitudinal oscillations of the Fairchild XR2K-1 airplane with weighted elevator. Aerodynamic balance gradually increased in successive runs. Speed 120 mph; power on.